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Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition

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Abstract We consider two game-theoretic models of the generation capacity expansion problem in liberalized electricity markets. The first is an open loop equilibrium model, where generation companies simultaneously choose capacities and quantities to maximize their individual profit. The second is a closed loop model, in which companies first choose capacities maximizing their profit anticipating the market equilibrium outcomes in the second stage. The latter problem is an Equilibrium Problem with Equilibrium Constraints (EPEC). In both models, the intensity of competition among producers in the energy market is frequently represented using conjectural variations. Considering one load period, we show that for any choice of conjectural variations ranging from Bertrand to Cournot, the closed loop equilibrium coincides with the Cournot open loop equilibrium, thereby extending the findings of Kreps and Scheinkman. When expanding the model framework to multiple load periods, the closed loop equilibria for different conjectural variations can diverge from each other and from open loop equilibria. Surprisingly, the rank ordering of the closed loop equilibria in terms of consumer surplus and market equilibria is ambiguous. Thus, regulatory approaches that force marginal cost-based bidding in spot markets may diminish market efficiency and consumer welfare by dampening incentives for investment.

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1 Introduction

In this paper we compare game-theoretic models that can be used to analyze the strategic behavior of companies facing generation capacity expansion decisions in liberalized electricity markets. Game theory is particularly useful in the restructured energy sector because it allows us to investigate the strategic behavior of agents (generation companies) whose interests are opposing and whose decisions affect each other. In particular, we seek to characterize the difference between open and closed loop models of investment.

Open loop models extend short-term models to a longer time horizon by modeling investment and production decisions as being taken at the same time. This corresponds to the open loop Cournot equilibrium conditions presented in [25], the Cournot-based model presented in [34], which is solved using a Mixed Complementarity Problem (MCP) scheme, and the model analyzed in [7], which is solved using an equivalent optimization problem. However, this approach may overly simplify the dynamic nature of the problem, as it assumes that expansion and operation decisions are taken simultaneously.

The reason to employ more complicated closed loop formulations is that the generation capacity expansion problem has an innate two-stage structure: first investment decisions are taken followed by determination of energy production in the spot market, which is limited by the previously chosen capacity. A two-stage decision structure is a natural way to represent how many organizations actually make decisions. One organizational subunit is often responsible for capital budgeting and anticipating how capital expenditures might affect future revenues and costs over a multi-year or even multi-decadal time horizon, whereas a different group is in charge of day-to-day spot market bidding and output decisions. This type of closed loop model is in fact an Equilibrium Problem with Equilibrium Constraints (EPEC), see [23,33], arising when each of two or more companies simultaneously faces its own profit maximization problem modeled as a Mathematical Program with Equilibrium Constraints (MPEC). In the electricity sector, MPECs, bilevel problems, and EPECs were first used to represent short-run bidding and production games among power producers with existing capacity, e.g.,[3,5,16,35,37]. EPECs belong to a recently developed class of mathematical programs that often arise in engineering and economics applications and can be used to model electricity markets [30]. For methods to solve EPECs, i.e., diagonalization, the reader is referred to [17,18,23].

1.1 Review of Literature

Several closed loop approaches to the generation capacity expansion problem have been proposed. The papers most relevant for our paper are [25] and [20], which will be discussed below. With their paper [20], Kreps and Scheinkman (K-S) tried to reconcile Cournot's [8] and Bertrand's [4] theory by constructing a two-stage game, where first firms simultaneously set capacity and second, after capacity levels are made public, there is price competition. They find that when assuming two identical firms and an efficient rationing rule (i.e., the market's short-run production is provided at least cost), their two-stage game yields Cournot outcomes. Davidson and Deneckere [9] formulate a critique of K-S, where they say that results critically depend on the choice of the rationing rule. They claim that if the rationing rule is changed, the equilibrium outcome need not be Cournot. In defense of K-S, Paul Madden proved [24] that if it is assumed that demand functions are of the constant elasticity form and that all costs are sunk, then the K-S two-stage game reduces to the Cournot model for any rationing mechanism between the efficient and proportional extremes. However, Deneckere and Kovenock [12] find that the K-S result does not necessarily hold if costs are asymmetric.

More recently, works such as [15,21] address the extension of the K-S model to uncertainty of marginal costs. [15] shows that due to uncertainty of marginal costs, equilibria were necessarily asymmetric. Reynolds and Wilson [31] address the issue of uncertain demand in a K-S like model, which is related to our extension to multiple load periods. They discover that if costs are sufficiently high, the Cournot outcome is the unique solution to this game. However, they also find that if costs are lower, no pure strategy equilibria exists. Lepore [22] also demonstrates that, under certain assumptions, the K-S result is robust to demand uncertainty. Our results extend the literature on K-S models by considering generalizations to conjectural variations other than Competitive (Bertrand) as well as multiple load periods or, equivalently, stochastic load.

In [25] the authors present and analyze three different models: an open loop perfectly competitive model, an open loop Cournot model and a closed loop Cournot model. Each considers several load periods which have different demand curves and two firms, one with a peak load technology (low capital cost, high operating cost) and the other with a base load technology (high capital cost, low operating cost). They analyze when open and closed loop Cournot models coincide and when they are necessarily different. Moreover, they demonstrate that the closed loop Cournot equilibrium capacities fall between the open loop Cournot and the open loop competitive solutions. Our paper differs by considering players and a range of conjectural variations between Bertrand (perfect competition) and Cournot. Our formal results are for symmetric agents but these results extend to asymmetric cases. We derive certain equivalency results that can also be extended to asymmetric firms.

In addition to [26], there are other works that have formulated and solved closed loop models of power generation expansion. In [34] we find a closed loop Stackelberg-based model that is formulated as an MPEC, where in the first stage a leader firm decides its capacity and then in the second stage the followers compete in quantities in a Cournot game. This work focuses on comparing numerical results between this Stackelberg model and an open loop Cournot model. [7] presents a two-stage model representing the market equilibrium, where the first stage is based on a Cournot equilibrium among producers who can choose continuous capacity investments and computes a market equilibrium approximation for the entire model horizon and a second stage discretizes this solution separately for each year. In [14] the authors present a linear bilevel model that determines the optimal investment decisions of one generation company. They consider uncertainty in the demand and in the capacity decisions of the competition. In [32] the author applies a two-stage model in which firms choose their capacities under demand uncertainty prior to competing in prices and presents regulatory conclusions. An instance of a stochastic static closed loop model for the generation capacity problem for a single firm can be found in [19], where investment and strategic production decisions are taken in the upper level for a single target year in the future, while the lower level represents market clearing where rival offering and investments are represented via scenarios and which maximizes social welfare.

Existing generation capacity expansion approaches in the literature assume either perfectly competitive [14] or Cournot behavior [34, 7] in the spot market. The proposed open and closed loop models of this paper extend previous approaches by including a generalized representation of market behavior via conjectural variations, in particular through an equivalent conjectured price response. This allows us to represent various forms of oligopoly, ranging from Bertrand to Cournot. Power market oligopoly models have been proposed before based on conjectural variations [6] and conjectured price responses [11], but only for short term markets in which capacity is fixed.

1.2 Open loop versus Closed loop Capacity Equilibria

We consider two identical firms with perfectly substitutable products, each facing either a one-stage or a two-stage competitive situation. The one-stage situation, represented by the open loop model, describes the one-shot investment operation market equilibrium. The closed loop model, which is an EPEC, describes the two-stage investment-operation market equilibrium. Considering one load period, we find that the closed loop equilibrium for any strategic market behavior between Bertrand and Cournot yields the open loop Cournot outcomes, thereby extending the result of K-S [20]. As previously mentioned, Murphy and Smeers [25] have found that under certain conditions the open and closed loop Cournot equilibria coincide. Our result furthermore shows that considering one load period, all closed loop models assuming strategic spot market behavior between Bertrand and Cournot coincide with the open loop Cournot solution. In the multiple load period case we define some sufficient conditions for the open and closed loop capacity decisions to be the same. However, this result is parameter dependent. When capacity is the same, outputs in non-binding load periods are the same for open and closed loop models when strategic spot market behavior is the same, otherwise outputs can differ.

When the closed loop capacity decisions differ for different conjectural variations, then the resulting consumer surplus and market efficiency (as measured by total surplus) will depend on the conjectural variation. It turns out that which conjectural variation results in the highest efficiency is parameter dependent. In particular, under some assumptions, the closed loop model considering Bertrand (perfect) competition in the energy market can actually result in lower market efficiency, lower consumer surplus and higher prices than Cournot competition. This surprising result implies that regulatory approaches that force marginal cost-based bidding in spot markets may decrease market efficiency and consumer welfare and may therefore actually be harmful. For example, the Irish spot market rules [29] require bids to equal short-run marginal cost. Meanwhile, local market power mitigation procedures in several US organized markets reset bids to marginal cost (plus a small adder) if significant market power is present in local transmission-constrained markets [28]. These market designs implicitly assume that Bertrand competition is welfare superior to more oligopolistic behavior, such as Cournot competition. As our counter-example will show, this is not necessarily so.

The results obtained are suggestive of what might occur in other industries where storage is relatively unimportant and there is time varying demand that must be met by production at the same moment. Examples include, for instance, industries such as airlines or hotels.

This paper is organized as follows. In section 2 we introduce and define the conjectured price response representation of the short-term market and provide a straight-forward relationship to conjectural variations. Then, in section 3 we formulate symmetric open and closed loop models for one load period and establish that K-S also holds for arbitrary strategic behavior ranging from Bertrand to Cournot competition. This is followed by section 4, which extends the symmetric K-S framework to multiple load periods. In section 5 we show by example that under the closed loop framework, more competitive behavior in the spot market can lead to less market efficiency and consumer surplus. Finally, section 6 concludes the paper.

2 Conjectural Variations and Conjectured Price Response

We introduce equilibrium models that capture various degrees of strategic behavior in the spot market by introducing conjectural variations into the short-run energy market formulation. The conjectural variations development is consistent with standard industrial organization theory [13]. In particular, we introduce a conjectured price response parameter that can easily be translated into conjectural variations with respect to quantities, and vice versa.

First, we consider two identical firms with perfectly substitutable products, for which we furthermore assume an affine price function $p(d)$, i.e., $p(d) = (D_0 - d)/\alpha$, where d is the quantity demanded, $\alpha = D_0/P_0$ is the demand slope, D_0 the demand intercept, and P_0 is the price intercept. Demand d and quantities produced q_i, q_{-i}, with i and $-i$ being the indices for the market agents, are linked by the market clearing condition $q_i + q_{-i} = d$. Hence, we will refer to price also as $p(q_i, q_{-i})$.

Then we define the conjectural variation parameters as $\Phi_{-i,i}$. These represent agent i's belief about how agent $-i$ changes its production in response to a change in i's production. Therefore:

$$\Phi_{-i,i} = \frac{dq_{-i}}{dq_i}, \quad i \neq -i, \tag{1}$$

$$\Phi_{i,i} = 1. \tag{2}$$

And hence using (1)-(2) and our assumed $p(q_i, q_{-i})$, we obtain:

$$\frac{dp(q_i, q_{-i})}{dq_i} = -\frac{1}{\alpha} \sum_{-i} \Phi_{-i,i} = -\frac{1}{\alpha}(1 + \sum_{-i \neq i} \Phi_{-i,i}) \tag{3}$$

As we are considering two identical firms in the models of this paper, we can assume that $\Phi_{i,-i} = \Phi_{-i,i}$ which we define as Φ and therefore relation (3) simplifies to:

$$\frac{dp(q_i, q_{-i})}{dq_i} = -\frac{1}{\alpha}(1 + \Phi) \tag{4}$$

Now let us define the conjectured price response parameter θ_i as company i's belief concerning its influence on price p as a result of a change in its output q_i:

$$\theta_i := -\frac{dp(q_i, q_{-i})}{dq_i} = \frac{1}{\alpha}(1 + \Phi) \geq 0, \tag{5}$$

which immediately shows how to translate a conjectural variations parameter into the conjectured price response and vice versa. The nonnegativity of (5) comes from the assumption that the conjectural variations parameter $\Phi \geq -1$. Throughout the paper we will formulate the equilibrium models using the conjectured price response parameter as opposed to the conjectural variations parameter, because its depiction of the firms' influence on price is more convenient for the derivations, as opposed to a firm's influence on production by competitors.

As has been proven in [10], this representation allows us to express special cases of oligopolistic behavior such as Bertrand, perfect competition, the Cournot oligopoly, or collusion. A general formulation of each firm's profit objective would state that $p = p(q_i, q_{-i})$, with the firm anticipating that price will respond to the firm's output decision. We term this the conjectured price response model. If the firm takes p as exogenous (although it is endogenous to the market), the result is the price-taking or perfect competition also called the Bertrand conjecture [20] (and $\Phi = -1$). Then the conjectured price response parameter θ_i equals 0, which means that none of the competing firms believes it can influence price (and $\Phi = -1$). If instead $p(q_i, q_{-i})$ is the inverse demand curve $D_0/\alpha - (q_i + q_{-i})/\alpha$, with q_{-i} being the rival firm's output which is taken as exogenous by firm i, then the model is a Nash-Cournot oligopoly. In the Cournot case, θ_i equals $1/\alpha$, which would translate to $\Phi = 0$ in the conjectural variations framework.

We can also express collusion (quantity matching, or 'tit for tat') as $2/\alpha$, which translates to $\Phi = 1$, as well as values between the extremes of perfect competition and the Cournot oligopoly. Intermediate values of Φ (or θ_i respectively) can be the reduced form result of more complex dynamic games. For example, Murphy and Smeers [26] show that the Allaz-Vila [1] two stage forward contracting/spot market game can be reduced to a one stage game assuming $\Phi = 1/2$ (or $\theta = 1/(2\alpha)$). The two stage Stackelberg game can also be reduced to a conjectural variations one stage game.

3 Generalization of the K-S Single Load Period Result to Arbitrary Oligopolistic Conjectures

In this section we consider two identical firms with perfectly substitutable products, facing either a one-stage or a two-stage competitive situation. The one-stage situation is represented by the open loop model presented in 3.1 and describes the one-shot investment-operation market equilibrium. In this situation, firms simultaneously choose capacities and quantities to maximize their individual profit, while each firm conjectures a price response to its output decisions consistent with the conjectured price response model. The closed loop model given in 3.2 describes the two-stage investment-operation market equilibrium, where firms first choose capacities that maximize their profit while anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured price response market equilibrium. We furthermore assume that there is an affine relation between price and demand and that capacity can be added in continuous amounts.

The main contribution of this section is Theorem 1, in which we show that for two identical agents, one load period and an affine non-increasing inverse demand function, the one-stage model solution assuming Cournot competition is a solution to the closed loop model independent of the choice of conjectured price response within the Bertrand-Cournot range. When the conjectured price response represents perfect competition, then this result is equivalent to the finding of [20]. Thus, Theorem 1 extends the Kreps and Scheinkman result to any conjectured price response within a range. Later in the paper, however, we show that this result does not generalize to the case of multiple load periods.

Throughout this section we will use the following notation: x_i denotes the capacity [MW] of firms $i = 1, 2$. q_i denotes the quantity [MW] produced by firms $i = 1, 2$. t [h/year] corresponds to the duration of the load period per year, p [€/MWh] denotes the clearing price, δ [€/MWh] is the variable production cost and β [€/MW/year] corresponds to the annual investment cost. Both cost terms are assumed to be nonnegative and we assume that the variable cost will be no more than the price intercept, i.e., $\delta \leq P_0$. d denotes quantity demanded [MW]. The same demand curve assumptions are made as in section

2. Finally, we consider one year rather than a multi-year time horizon, and so each firm is maximizing its annualized profit.

3.1 The Open Loop Model

In the open loop model, every firm i faces a profit maximization problem in which it chooses capacity x_i and production q_i simultaneously. When firms simultaneously compete in capacity and quantity, the open loop investment-operation market equilibrium problem consists of all the firms' profit maximization problems plus market clearing conditions that link together their problems by $d = D_0 - \alpha p(q_i, q_{-i})$. Conceptually, the resulting equilibrium problem can be written as (6) - (7):

$$\forall i \begin{cases} \max_{x_i,q_i} \quad t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \qquad\qquad s.t. \quad q_i \leq x_i \end{cases} \tag{6}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \tag{7}$$

In (6) we describe i's profit maximization as consisting of market revenues $tp(q_i, q_{-i})q_i$ minus production costs $t\delta q_i$ and investment costs βx_i. Although (6)'s constraint is expressed as an inequality, it will hold as an equality in equilibrium, at least in this one-period formulation. That $x_i = q_i$ for $i = 1, 2$ will be true in equilibrium, can easily be proven by contradiction. Let us assume that at the equilibrium $x_i > q_i$; then firm i could unilaterally increase its profits by shrinking x_i to q_i (assuming $\beta > 0$), which contradicts the assumption of being at an equilibrium.

In this representation the conjectured price response is not explicit. Therefore we re-write the open loop equilibrium stated in (6) - (7) as a Mixed Complementarity Problem (MCP) by replacing each firm's profit maximization problem by its first order Karush-Kuhn-Tucker (KKT) conditions. Therefore let $\mathcal{L}_i$ denote the Lagrangian of company i's corresponding optimization problem, given in (6) and let λ_i be the Lagrange multiplier of constraint $q_i \leq x_i$. Then under the assumption that the equilibrium is nontrivial $(q_i, x_i > 0)$, the open loop equilibrium problem is then given in (8)-(9).

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = tp(q_i, q_{-i}) - t\theta q_i - t\delta - \lambda_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = \beta - \lambda_i = 0 \\ q_i \leq x_i \\ \lambda_i \geq 0 \\ \lambda_i(x_i - q_i) = 0 \end{cases} \tag{8}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \tag{9}$$

Due to the fact that $\lambda_i = \beta > 0$, the complementarity condition yields $x_i = q_i$ in equilibrium. In this formulation we can directly see the conjectured price response parameter θ in $\frac{\partial \mathcal{L}_i}{\partial q_i}$. Solving the resulting system of equations yields:

$$q_i = \frac{D_0 t - \alpha(\beta + \delta t)}{t(\alpha\theta + 2)} \quad \forall i \tag{10}$$

$$p = \frac{D_0 t\theta + 2(\beta + \delta t)}{t(\alpha\theta + 2)}. \tag{11}$$

The open loop model has a non-trivial solution if data is chosen such that $D_0 t - \alpha(\beta + \delta t) > 0$ is satisfied. Otherwise the solution will be the trivial one.

A special case of the conjectured price response is the Cournot oligopoly. In order to obtain the open loop Cournot solution, we just need to insert the appropriate value of the conjectured price response parameter θ, which for Cournot is $\theta = 1/\alpha$. This solution is unique [25]. Then (10)-(11) yield:

$$q_i = \frac{D_0 t - \alpha(\beta + \delta t)}{3t} \quad \forall i \tag{12}$$

$$p = \frac{D_0 t + 2\alpha(\beta + \delta t)}{3t\alpha}. \tag{13}$$

3.2 The Closed Loop Model

We now present the closed loop conjectured price response model describing the two-stage investment-operation market equilibrium. In this case, firms first choose capacities maximizing their profit anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured price response market equilibrium. We stress that the main distinction of this model from the equilibrium model described in section 3.1 is that now there are two stages in the decision process, i.e., capacities and quantities are not chosen at the same time. Then we present Theorem 1 which establishes a relation between the open loop and the closed loop models for the single demand period case.

3.2.1 The Production Level - Second Stage

The second stage (or lower level) represents the conjectured-price-response market equilibrium, in which both firms maximize their market revenues minus their production costs, deciding their production subject to the constraint that production will not exceed capacity. The argument given above shows, at equilibrium, that this constraint binds if there is a single demand period. These maximization problems are linked by the market clearing condition. Thus, the second stage market equilibrium problem can be written as:

$$\forall i \begin{cases} \max_{q_i} & t(p(q_i, q_{-i}) - \delta)q_i \\ & s.t. \quad q_i \le x_i \end{cases} \tag{14}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}), \tag{15}$$

As in the open loop case, p may be conjectured by firm i to be a function of its output q_i. We now substitute firm i's KKT conditions for (14) and arrive at the the conjectured price response market equilibrium conditions given by:

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = tp(q_i, q_{-i}) - t\theta q_i - t\delta - \lambda_i = 0 \\ q_i \leq x_i \\ 0 \leq \lambda_i \\ \lambda_i (x_i - q_i) = 0 \end{cases} \tag{16}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \tag{17}$$

Again, we assume that the solution is nontrivial ($q_i > 0$).

3.2.2 The Investment Level - First Stage

In the first stage, both firms maximize their total profits, consisting of the gross margin from the second stage (revenues minus variable production costs) minus investment costs, and choose their capacities subject to the second stage equilibrium response. This can be written as the following equilibrium problem:

$$\forall i \begin{cases} \max_{x_i} \quad t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ s.t. \quad \text{Second Stage, } (16) - (17) \end{cases} \tag{18}$$

We know that at equilibrium, production will be equal to capacity. As in the open loop model, this can be shown by contradiction. Since there is a linear relation between price and demand, it follows that price can be expressed as $p = \frac{D_0 - d}{\alpha}$. Substituting $x_i = q_i$ in this expression of price, yields $p = \frac{D_0 - x_1 - x_2}{\alpha}$. Then expressing the objective function and the second stage in terms of the variables x_i yields the following simplified closed loop equilibrium problem:

$$\forall i \begin{cases} \max_{x_i} \quad t(\frac{D_0 - x_1 - x_2}{\alpha} - \delta)x_i - \beta x_i \\ s.t. \quad \frac{D_0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \geq 0 \quad : \gamma_i \end{cases} \tag{19}$$

where γ_i are the dual variables to the corresponding constraints. Writing down the closed loop equilibrium conditions (assuming a nontrivial solution $x_i > 0$) then yields:

$$\forall i \begin{cases} t(\frac{D_0 - x_1 - x_2}{\alpha} - \delta) - tx_i/\alpha - \beta + \gamma_i(-\theta - 1/\alpha) = 0 \\ \frac{D_0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \geq 0 \\ \gamma_i(\frac{D_0 - x_1 - x_2}{\alpha} - \theta x_i - \delta) = 0 \\ \gamma_i \geq 0 \end{cases} \tag{20}$$

When solving the system of equations given by (20) we distinguish between two separate cases: $\gamma_i = 0$ and $\gamma_i > 0$. The first case, i.e. $\gamma_i = 0$, yields the following solution for the closed loop equilibrium, where λ_i has been obtained from (16):

$$x_i = \frac{D_0 t - \alpha(\beta + \delta t)}{3t} \quad \forall i. \tag{21}$$

$$p = \frac{D_0 t + 2\alpha(\beta + \delta t)}{3t\alpha} \tag{22}$$

$$\lambda_i = \frac{D_0 t + \alpha^2(\beta + \delta t)\theta + \alpha(2\beta - t(\delta + D_0\theta))}{3\alpha} \quad \forall i. \tag{23}$$

Moreover, it is easy to show that for $\theta \in [0, 1/\alpha]$ $\lambda_i \geq 0$ will be satisfied, [1] which shows that x_i is indeed the optimal value of q_i in (16), confirming the validity of (19) for $\gamma_i = 0$.

As for uniqueness of the closed loop equilibrium, [25] has proven for the Cournot closed loop equilibrium that if an equilibrium exists, then it is unique. We will investigate uniqueness issues of the closed loop conjectured price response model in future research. Comparing (21) and (22) with the open loop equilibrium (10) and (11) we see that this is exactly the open loop solution considering Cournot competition, i.e. (12) and (13).

Now let us consider the second case, i.e., $\gamma_i > 0$. Then (20) yields the following values for capacities and γ_i:

$$x_i = \frac{D_0 - \alpha\delta}{2 + \alpha\theta} \quad \forall i. \tag{24}$$

$$\gamma_i = \frac{-(D_0 t + \alpha^2(\beta + \delta t)\theta + \alpha(2\beta - t(\delta + D_0\theta)))}{(\alpha\theta + 1)(\alpha\theta + 2)} \quad \forall i. \tag{25}$$

In the formulation of γ_i in (25), the numerator of the right hand side is the negative of numerator on the right hand side of the formula (23), where we know that latter is nonnegative for $\theta \in [0, 1/\alpha]$. That is, it is impossible for $\gamma_i > 0$. Hence the only solution to the closed loop equilibrium is the open loop Cournot solution that results when $\gamma_i = 0$.

3.2.3 Theorem 1

Theorem 1. *Let there be two identical firms with perfectly substitutable products. Moreover, let price p be an affine function of demand d, i.e.,* $p = (D_0 - d)/\alpha p$*, and let there be one load period. Then the open loop Cournot solution, see* (12)-(13)*, is a solution to the closed loop conjectured price response equilibrium* (21)-(23) *for any choice of the CPR parameter* θ *from Bertrand to Cournot competition.*

Proof : Sections 3.1 and 3.2 above prove this theorem. As in the open loop model, the closed loop model has a non-trivial solution if data is chosen such that $D_0 t - \alpha(\beta + \delta t) > 0$ is satisfied. Otherwise the solution will be the degenerate solution $q_i = 0$, $x_i = 0$, $d = 0$, $p = D_0/\alpha$. □

Theorem 1 extends to the case of asymmetric firms but we omit the somewhat tedious analysis which can, however, be found in [36].

What we have proven in Theorem 1 is that as long as the strategic behavior in the market (which is characterized by the parameter θ) is more competitive

[1] Case $\theta = 0$: from (23) we get $D_0 t + 2\alpha\beta - \alpha\delta t = D_0 P_0 t + 2\alpha\beta P_0 - D_0\delta t \geq 2\alpha\beta P_0 \geq 0$; Case $\theta = 1/(k\alpha)$ with $k \geq 1$: $D_0 t + \alpha^2(\beta+\delta t)/(k\alpha) + \alpha(2\beta - t(\delta + D_0(k\alpha))) = (k-1)D_0 t/k + 2\alpha\beta + \alpha\beta/k - (k-1)D_0 t\delta/(kP_0) \Rightarrow (k-1)D_0 tP_0/k + 2\alpha\beta P_0 + \alpha\beta P_0/k - (k-1)D_0 t\delta/k \geq 2\alpha\beta P_0 + \alpha\beta P_0/k \geq 0$.

than Cournot, then in the closed loop problem firms will always decide to build Cournot capacities. Even when the market is more competitive than the Cournot case (e.g., Allaz-Vila or Bertrand), firms will build Cournot capacities. Hence Theorem 1 states that the Kreps and Scheinkman result holds for any conjectured price response more competitive than Cournot (e.g., Allaz-Vila or Bertrand), not just for the case of Bertrand second stage competition.

Note that Theorem 1 describes sufficient conditions but they are not necessary. This means that there are cases where Theorem 1 also holds for $\theta > 1/\alpha$.

For example Theorem 1 may hold under collusive behavior ($\theta = 2/\alpha$) when the marginal cost of production (δ) is sufficiently small.[2]

In the following section we will extend the result of Theorem 1 to the case of multiple demand periods. In particular, under stringent conditions, the Cournot open loop and closed loop solutions can be the same, and the Cournot open loop capacity can be the same as the closed loop capacity for more intensive levels of competition in the second stage of the closed loop game. But this result is parameter dependent, and in general, these solutions differ. Surprisingly, for some parameter assumptions, more intensive competition in the second stage can yield economically inferior outcomes compared to Cournot competition, in terms of consumer surplus and total market surplus.

4 Extension of K-S Result for Multiple Load Periods

In this section we extend the previously established comparison between the open loop and the closed loop model to the situation in which firms each choose a single capacity level, but face time varying demand that must be met instantaneously. This characterizes electricity markets in which all generation capacity is dispatchable thermal plant and there is no significant storage (e.g., in the form of hydropower). We also do not consider intermittent nondispatchable resources (such as wind); however, if their capacity is exogenous, their output can simply be subtracted from consumer quantity demanded, so that d represents effective demand. In particular, this extension will be characterized by Proposition 1.

As in the previous section we still consider two identical firms and a linear demand function. Additionally let us define l as the index of different load periods. Now production decisions depend on both i and l. Furthermore let us define the active set of load levels LB_i as the set of load periods in which equilibrium productions equals capacity for firm i, i.e., $LB_i := \{l | q_{il} = x_i\}$.

Proposition 1. *(a) If the closed loop solutions for different θ between Bertrand and Cournot competition have the same active set of load periods (i.e., firm i's upper bound on production is binding for the same load periods l) and the second stage multipliers, corresponding to the active set, are positive at equilibrium, then capacity x_i is the same for those values of θ. (b) Furthermore, if*

[2] Let $D_0 = 1, t = 1, \alpha = 1, \beta = 1/2$ and $\delta = 0$, then the open loop Cournot solution is $p = 2/3$, with $x = 1/6$ for each firm. In this case, with these cost numbers, the open loop Cournot equals the closed loop equilibrium with $\theta = 2/\alpha$ (collusion, $\Phi = 1$).

we assume that the open loop Cournot equilibrium, i.e., $\theta = 1/\alpha$, has the same active set, then the Cournot open and closed loop equilibria are the same.

4.1 The Open Loop Model for Multiple Load Periods

The purpose of this section is to develop the stationary conditions for the open loop model for general θ and multiple load periods and thereby characterize the equilibrium capacity x_i. Therefore, we write the open loop investment-operation market equilibrium as:

$$\forall i \begin{cases} \max_{x_i,q_{il}} & \sum_l t_l(p_l(q_{il}, q_{-il}) - \delta)q_{il} - \beta x_i \\ & s.t. \quad q_{il} \leq x_i \quad \forall l \end{cases} \tag{26}$$

$$d_l = q_{il} + q_{-il}, \quad d_l = D_{0l} - \alpha_l p_l(q_{il}, q_{-il}) \quad \forall l \tag{27}$$

Let us now derive the investment-operation market equilibrium conditions distinguishing load levels where capacity is binding from when capacity is slack. We can omit the complementarity between λ_{il} and $q_{il} < x_i$, because $\lambda_{il} = 0$ for $l \notin LB_i$ and $x_i = q_{il}$ for $l \in LB_i$.

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = t_l p_l(q_{il}, q_{-il}) - t_l\theta_l q_{il} - t_l\delta - \lambda_{il} = 0 \quad l \in LB_i \\ \frac{\partial \mathcal{L}_i}{\partial q_{il}} = t_l p_l(q_{il}, q_{-il}) - t_l\theta_l q_{il} - t_l\delta = 0 \quad l \notin LB_i \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = -\beta + \sum_{l \in LB_i} \lambda_{il} = 0 \\ q_{il} = x_i \quad l \in LB_i \\ q_{il} < x_i \quad l \notin LB_i \\ 0 \leq \lambda_{il} \quad \forall l \end{cases} \tag{28}$$

$$d_l = q_{il} + q_{-il}, \quad d_l = D_{0l} - \alpha_l p_l(q_{il}, q_{-il}) \quad \forall l \tag{29}$$

(As in section 3, we assume nontrivial equilibrium in which $q_{il} > 0$). For the non-binding load periods $l \notin LB_i$ we can obtain the solution to the equilibrium by solving the system of equations given by (28)-(29), which yields:

$$q_{il} = \frac{D_{0l} - \alpha_l\delta}{2 + \alpha_l\theta_l} \quad \forall i, l \notin LB_i \tag{30}$$

$$p_l = \frac{D_{0l}\theta + 2\delta}{2 + \alpha_l\theta_l} \quad \forall l \notin LB_i. \tag{31}$$

In order to obtain the solution for load levels when capacity is binding, we sum $\frac{\partial \mathcal{L}_i}{\partial q_{il}}$ over all load periods $l \in LB_i$, substitute $q_{il} = x_i$ and use the $\frac{\partial \mathcal{L}_i}{\partial x_i} = 0$ condition:

$$\sum_{l \in LB_i} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = \sum_{l \in LB_i} (t_l p_l(q_{il}, q_{-il}) - t_l\delta - t_l\theta_l q_{il}) - \sum_{l \in LB_i} \lambda_{il} \tag{32}$$

$$= \sum_{l \in LB_i} (t_l p_l(q_{il}, q_{-il}) - t_l\delta - t_l\theta_l x_i) - \beta = 0 \tag{33}$$

If we express price as a function of capacity ($q_i = x_i$) and we solve the system of equations (29), together with (33) $\forall i$, this yields:

$$x_i = \frac{\sum_{l\in LB_i}(D_{0l}t_l \prod_{n\neq l\in LB_i}\alpha_n) - \prod_{l\in LB_i}\alpha_l(\beta + \delta\sum_{l\in LB_i}t_l)}{\sum_{l\in LB_i}(t_l(2+\alpha_l\theta_l)\prod_{n\neq l\in LB_i}\alpha_n)} \quad ,\forall i \quad (34)$$

We know that for $\theta_l \in [0, 1/\alpha_l]$, q_{il} will be a continuous function of x_i and hence from (28) we get that λ_{il} will also be a continuous function of x_i. Having obtained capacities x_i, the prices p_l and demand d_l for $l \in LB_i$ follow.

Above it has not been explicitly stated that q_i and x_i are positive variables, but it can be seen that this is satisfied at the equilibrium point as long as $\sum_{l\in LB_i}(D_{0l}t_l\prod_{n\neq l\in LB_i}\alpha_n) > \prod_{l\in LB_i}\alpha_l(\beta+\delta\sum_{l\in LB_i}t_l)$ holds. If this does not hold, then the equilibrium automatically becomes the degenerate case $q_{il} = 0$, $x_i = 0$, $d_l = 0$, $p_l = D_{0l}/\alpha_l$.

4.2 The Closed Loop Model for Multiple Load Periods

Let us now derive the stationary conditions for the closed loop problem for multiple load periods which then yields an expression for the equilibrium capacity. First, we state the second stage production game for the closed loop game with multiple load periods in (35)-(36) and define Lagrange multipliers λ_{il} for the constraint $q_{il} \leq x_i$.

$$\forall i \begin{cases} \max_{q_{il}} & \sum_l t_l(p_l(q_{il}, q_{-il}) - \delta)q_{il} \\ & s.t. \quad q_{il} \leq x_i \quad \forall l \end{cases} \quad (35)$$

$$d_l = q_{il} + q_{-il}, \quad d_l = D_{0l} - \alpha_l p_l(q_{il}, q_{-il}) \quad \forall l \quad (36)$$

Now we derive the market equilibrium conditions, assuming that each firm holds the same conjectured price response θ_l in each load period l. θ_l can differ among periods. The complementarity between λ_{il} and $q_i < x_i$ for $l \notin LB_i$ implies that $\lambda_{il} = 0$ for $l \notin LB_i$. Hence we omit that complementarity condition for those load periods in the market equilibrium formulation of (37)-(38). Moreover, we assume that multipliers λ_{il} for $l \in LB_i$ will be positive at equilibrium. (If any multipliers are zero, then Proposition 1 may not hold.)

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = t_l p_l(q_{il}, q_{-il}) - t_l\theta_l q_{il} - t_l\delta - \lambda_{il} = 0 & l \in LB_i \\ \frac{\partial \mathcal{L}_i}{\partial q_{il}} = t_l p_l(q_{il}, q_{-il}) - t_l\theta_l q_{il} - t_l\delta = 0 & l \notin LB_i \\ q_{il} = x_i & l \in LB_i \\ q_{il} < x_i & l \notin LB_i \\ 0 \leq \lambda_{il} & \forall l \end{cases} \quad (37)$$

$$d_l = q_{il} + q_{-il}, \quad d_l = D_{0l} - \alpha_l p_l(q_{il}, q_{-il}) \quad \forall l \quad (38)$$

For the non-binding load periods $l \notin LB_i$ we can obtain the solution to the conjectured price response market equilibrium by solving the system of equations given by (37)-(38), which yields:

$$q_{il} = \frac{D_{0l} - \alpha_l\delta}{2 + \alpha_l\theta_l} \quad \forall i, l \notin LB_i \quad (39)$$

$$p_l = \frac{D_{0l}\theta + 2\delta}{2 + \alpha_l \theta_l} \quad \forall l \notin LB_i. \tag{40}$$

We cannot yet solve the market equilibrium for the binding load periods $l \in LB_i$ depending as they do upon the x_i's. Hence we move on to the investment equilibrium problem to obtain those x_i's, which is formulated below:

$$\forall i \begin{cases} \max_{x_i} & \sum_l t_l p_l(q_{il}, q_{-il}) q_{il} - \sum_l t_l \delta q_{il} - \beta x_i \\ & \qquad s.t. \quad (37) - (38) \end{cases} \tag{41}$$

After recalling that $q_i = x_i$ for l belonging to LB_i and then re-arranging terms, we can rewrite the objective function as:

$$\sum_{l \in LB_i} (t_l p_l x_i - t_l \delta x_i) + \sum_{l \notin LB_i} (t_l p_l q_{il} - t_l \delta q_{il}) - \beta x_i \tag{42}$$

Note that we have separated the terms of the objective function that correspond to inactive capacity constraints ($l \notin LB_i$) which do not involve x_i's at all, and the terms that correspond to the active capacity constraints ($l \in LB_i$). We furthermore know that for load periods $l \in LB_i$ the price $p_l = \frac{D_{0l} - d_l}{\alpha_l} = \frac{D_{0l} - \sum_i x_i}{\alpha_l}$. Replacing p_l for $l \in LB_i$ in (42), yields:

$$\sum_{l \in LB_i} (t_l \frac{D_{0l} - x_1 - x_2}{\alpha_l} x_i - t_l \delta x_i) + \sum_{l \notin LB_i} (t_l p_l q_{il} - t_l \delta q_{il}) - \beta x_i \tag{43}$$

We now show that (43) is valid for small perturbations of x_i around its equilibrium level. In other words, (43) is a local description of the MPEC (41). It has been shown in [2] that the second stage problem, i.e., the conjectured price response spot market equilibrium, has an equivalent strictly concave optimization problem. Then the solution q_{il} is unique [27]. This yields that q_{il} is a continuous function of x_i. Therefore it follows from uniqueness of multipliers as a function of the optimal second stage quantity (due to the linear independence constraint qualification [27]) that λ_{il} is also a continuous function of x_i. Hence, for small changes in x_i the active set will not change and we obtain smoothness of objective function (43).

Therefore we can take the derivative of the objective function (43) with respect to x_i, setting it to zero and solving for x_i, which yields:

$$x_i = \frac{\sum_{l \in LB_i} (D_{0l} t_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l (\beta + \delta \sum_{l \in LB_i} t_l)}{3 \sum_{l \in LB_i} (t_l \prod_{n \neq l \in LB_i} \alpha_n)} \quad \forall i \tag{44}$$

We observe that the capacity given by (44) is independent of θ_l. This means that for any other closed loop equilibrium whose active set coincides with LB_i and whose λ_{il} are positive at equilibrium, the capacity at equilibrium will also be described by (44), even though strategic spot market behavior may be different.

Now that we have obtained the values for x_i, the values for q_{il} as well as prices p_l and demand d_l with $l \in LB_i$ follow. The closed loop model only yields a non-trivial solution if data is chosen such that $\sum_{l \in LB_i} (D_{0l} t_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l (\beta + \delta \sum_{l \in LB_i} t_l) \geq 0$ is satisfied. Otherwise the solution will be the degenerate solution of zero capacities.

4.3 Proof of Proposition 1

Proof : *(Part a)* First, we observe that the closed loop capacity, given by (44), does not depend on the conjectured price responses θ_l, for $l = 1, \ldots, L$, and in particular this means that two closed loop equilibria with different θ_l's have the exact same capacity solution as long as their active sets are the same with λ_{il} positive for $l \in LB_i$.

(Part b) Comparing the closed loop capacity (44) with the open loop capacity (34) we note that the open loop capacity does depend on the strategic behavior θ_l in the market whereas the closed loop capacity does not. Moreover we observe that if open loop and closed loop models have the same active set at equilibrium, then their solutions are exactly the same under Cournot competition ($\theta_l = 1/\alpha_l$). If open and closed loop equilibria have the same active set and their θ_l coincide but are not Cournot, then in general their capacity will differ. However, their production q_{il} for $l \notin LB_i$ will be identical, as can be seen by comparing (30)-(31) and (39)-(40). $\square$

In general, prices will be lower in the second stage under Bertrand competition than under Cournot competition for periods other than LB_i. In that case, consumers will be better off (and firms worse off) under Bertrand competition than Cournot competition. The numerical example in section 4.4 illustrates this point. However, this result is parameter dependent as will be demonstrated by the example in section 5, where we will show that in some cases Cournot competition can yield more capacity and higher market efficiency than Bertrand competition. This can only occur for cases where either the binding LB_i differ, or the LB_i are the same but the λ_{il} are zero for some l. Note that for one load period, Proposition 1 reduces to Theorem 1.

Proposition1, like Theorem 1, can be extended to asymmetric firms. As the details are tedious we refer the reader to the general proof presented in [36].

4.4 Example with Two Load Periods: LB_i the same for all θ

Let us now consider an illustrative numerical example where two firms both consider an investment in power generation capacity that has an annualized capital cost $\beta = 46,000$ [€/MW/year], and operating cost $\delta = 11.8$ [€/MWh]. There are two load periods l, with durations of $t_1 = 3,760$ and $t_2 = 5,000$ hours per year, demand intercepts D_{0l} given by $D_{0,1} = 2,000$ and $D_{0,2} = 1,200$ MW, and demand slopes α_l equal to $\alpha_1 = D_{0,1}/250$ and $\alpha_2 = D_{0,2}/200$. Having chosen the demand data for the two load levels such that capacity will not be binding in both periods in any solution, we solve the open loop and the closed loop model and compare results. In Figure 1 we present the solution of one firm (as the second firm will have the same solution). First we depict the capacity that was built, then we compare production for both load periods and finally profits. Note that for both firms, LB_i will be the same for all θ and will include only period $l = 1$. Later we will present another example where this is not the case, and the results differ in important ways.

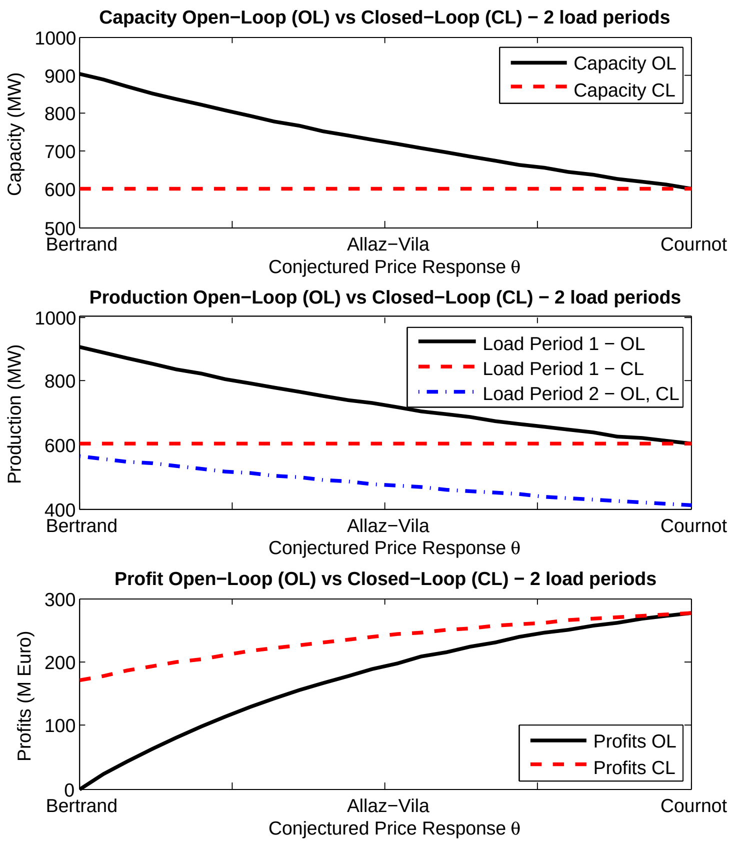

Fig. 1 Built capacity, production and profit of one firm in the two load period numerical example.

As demonstrated in Proposition 1, the closed loop capacity does not depend on behavior in the spot market. However we will see that profits do depend on the competitiveness of short-run behavior, and unlike the single demand period case, are not the same for all θ between Bertrand and Cournot. We refer to the binding load period as 'peak' and to the non-binding load period as 'base'. The closed loop production in the peak load level is the same for all θ, as long as the competitive behavior on the spot market is at least as competitive as Cournot. However, base load production depends on the strategic behavior in the spot market. This can be explained as follows: as long as the strategic behavior in the spot market is at least as competitive as Cournot, peak load outputs are independent of θ because agents are aware that building Cournot capacities will cause the peak period capacity constraint to bind and will limit production on the market to the Cournot capacity. However, given our demand data we also know that capacities will not be binding in the base period and as

Table 1 Closed Loop Equilibrium Solution Bertrand ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$) and Cournot($\theta_l = 1/\alpha_l$) second-stage competition.

l		1	2
q_{il}[MW]	$\theta_l = 0$	602.6	564.6
q_{il}[MW]	$\theta_l = 1/(2\alpha_l)$	602.6	451.7
q_{il}[MW]	$\theta_l = 1/\alpha_l$	602.6	376.4
p_l [€/MWh]	$\theta_l = 0$	99.4	11.8
p_l [€/MWh]	$\theta_l = 1/(2\alpha_l)$	99.4	49.4
p_l [€/MWh]	$\theta_l = 1/\alpha_l$	99.4	74.5

a consequence outputs will not be limited either. Hence during the base periods the closed loop model will find it most profitable to produce the equilibrium outcomes resulting from the particular conjectured price response.

On the other hand, when considering the open loop model, the capacity (peak load production) does depend on θ. In particular, the open loop capacity will be determined by the spot market equilibrium considering the degree of competitive behavior specified by θ. We observe that for increasing θ between Bertrand and Cournot in the open loop model, less and less capacity is built until we reach the Cournot case, at which point the open and closed loop results are exactly the same. Comparing open and closed loop models for a given θ reveals that while their base load outputs are identical, see Tables 1-2, capacity and thus peak load production differs depending on θ. Figure 1 also shows that profits obtained in the closed loop model equal or exceed the profits of the open loop model. This gap is largest assuming perfect competition and becomes continuously smaller for increasing θ until the results are equal under Cournot. This means that the further away that spot market competition is from Cournot, the greater the difference between model outcomes.

In standard open loop oligopoly models [13] without capacity constraints, Bertrand competition gives lower prices and total profits of firms, and greater consumer surplus, and market efficiency compared to Cournot competition.[3] We observe that this occurs for this particular instance of the open and closed loop models, see Tables 2 and 3. It can be readily proven more generally that market efficiency, consumer surplus, and average prices are greater for lower values of θ (more competitive second stage conditions) if LB_i are the same for those θ (and multipliers are positive), and capacity is not binding in every l.[4] However, we will also demonstrate by counter-example that this result does not necessarily apply when LB_i differ for different θ. In particular, in section 5 we will present an example in which Cournot competition counterintuitively yields higher market efficiency than Bertrand competition.

[3] Total Profit is defined as $\sum_l t_l(p_l - \delta)(q_{il} + q_{-il}) - \beta(x_i + x_{-i})$. Consumer Surplus (CS) is defined as the integral of the demand curve minus payments for energy, equal here to $\sum_l t_l(P_{0l} - p_l)(q_{il} + q_{-il})/2$. Market Efficiency (ME) is defined as CS plus Total Profits.

[4] This is proven by demonstrating that for smaller θ, the second stage prices will be lower and closer to marginal operating cost in load periods for those periods that capacity is not binding

Table 2 Open Loop Equilibrium Solution Bertrand ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$) and Cournot($\theta_l = 1/\alpha_l$) second-stage competition.

l		1	2
q_{il}[MW]	$\theta_l = 0$	903.9	564.6
q_{il}[MW]	$\theta_l = 1/(2\alpha_l)$	723.1	451.7
q_{il}[MW]	$\theta_l = 1/\alpha_l$	602.6	376.4
p_l [€/MWh]	$\theta_l = 0$	24.0	11.8
p_l [€/MWh]	$\theta_l = 1/(2\alpha_l)$	69.2	49.4
p_l [€/MWh]	$\theta_l = 1/\alpha_l$	99.4	74.5

Table 3 Market Efficiency (ME), Consumer Surplus (CS) and Total Profit in Closed Loop Solutions.

	Bertrand	Allaz-Vila	Cournot
ME [€]	$1.21 \cdot 10^9$	$1.19 \cdot 10^9$	$1.15 \cdot 10^9$
CS [€]	$0.873 \cdot 10^9$	$0.681 \cdot 10^9$	$0.577 \cdot 10^9$
Total Profit [€]	$0.342 \cdot 10^9$	$0.511 \cdot 10^9$	$0.578 \cdot 10^9$

5 Ambiguity in Ranking of Closed Loop Equilibria when LB_i Differs for Different θ

In this section we show by counter-example that the ranking of the closed loop conjectured price response equilibria, in terms of market efficiency (aggregate consumer surplus and market surplus) and consumer welfare, is parameter dependent. An interesting result we obtain is that it is possible for the closed loop model that assumes Bertrand competition (perfectly competitive behavior) in the market actually results in lower market efficiency (as measured by the sum of surpluses for all parties and load periods), lower consumer surplus, and higher average prices than when Cournot competition prevails. This counter-intuitive result implies that contrary to regulators' beliefs that requiring marginal cost bidding protects consumers, it actually can be harmful. Moreover, we observe that an intermediate solution between Bertrand and Cournot competition can lead to even higher installed capacity, larger social welfare and consumer surplus. In particular: *The ranking of conjectured price response equilibria in terms of market efficiency and consumer welfare is parameter dependent.* This occurs because in general the LB_i differ among the solutions. It does not occur when LB_i are the same for all θ and multipliers are positive as proven (and illustrated) in the previous section.

Proof by example: let us now consider two firms both making an investment in generation capacity having capital cost $\beta = 46,000$ [€/MW/year], and operating cost $\delta = 11.8$ [€/MWh]. There are twenty equal length load periods l, each with a duration of 438 hours per year ($t_l = 438$ for $l = 1, \ldots, 20$), the demand intercept D_{0l} of each load period can be obtained by $D_{0l} = 2,000 - 50(l - 1)$ for $l = 1, \ldots, 20$, and the demand slope α_l of each load period will be obtained by $D_{0l}/250$ for $l = 1, \ldots, 20$.

First we will assume Bertrand (perfect) competition, i.e., $\theta_l = 0$. We solve the resulting closed loop game by diagonalization [18], which is an iterative method in which firms take turns updating their first-stage capacity decisions, each time solving a two-stage MPEC while considering the competition's capacity decisions as fixed. The closed loop equilibrium solution assuming Bertrand competition is shown in Table 4. Second, we assume Cournot competition in the spot market, i.e., $\theta_l = 1/\alpha_l$. Again we solve the closed loop game by diagonalization, yielding the results shown in Table 5. We observe that under Bertrand competition, the capacity of 456.2 MW is binding in every load period and prices never fall to marginal operating cost. Moreover, the total installed capacity of 912.4 MW is significantly lower than that installed under Cournot, which is 1111.6 MW. On the other hand, under Cournot competition, each firm's capacity of 555.8 MW is binding only in the first six load periods and the firms exercise market power by restricting their output to below capacity in the other fourteen periods. Furthermore considering that the Cournot capacity is well above the Bertrand capacity, it follows that during the six peak load periods, Bertrand prices will be higher than Cournot prices.

This investment game can be viewed as a kind of prisoners' dilemma among multiple companies. An individual company might be able to unilaterally improve its profit by expanding capacity, with higher volumes making up for lower prices. But if all companies do that, then everyone's profits could be lower than if all companies instead refrained from building. (Of course, in this prisoners' dilemma metaphor we have not taken into account another set of players that is better off when the companies all build. These are the consumers, who enjoy lower prices and more consumption; as a result, overall market efficiency as measured by total market surplus may improve when firms "cheat".)

Standard (single stage) oligopoly models [13] without capacity constraints find that Bertrand competition gives lower prices and greater market efficiency than Cournot. Considering that standard result, our results seem counter-intuitive, but they are due to the two-stage nature of the game. In particular, less intensive competition in the commodity market can result in more investment and more consumer benefits than if competition in the commodity market is intense (price competition a la Bertrand). In terms of the prisoners' dilemma metaphor, higher short run margins under Cournot competition provide more incentive for the "prisoners" to "cheat" by adding capacity. Note that in order to get these counter-intuitive results, firms do not need to be symmetric, as shown in a numerical example in [36].

Finally, we solve the closed loop game assuming Allaz-Vila as competitive behavior between Bertrand and Cournot, i.e., $\theta_l = 1/(2\alpha_l)$. This yields the equilibrium given in Table 6. Comparing the market efficiency (ME) and the consumer surplus (CS) that we obtain in the Bertrand, Cournot and Allaz-Vila cases in Table 7, we observe that, surprisingly, the highest social welfare and the highest consumer surplus is obtained under the intermediate Allaz-Vila case. Even more surprising is that the capacity obtained under Allaz-Vila competition is lower than the Cournot capacity, but still yields a higher social welfare. This is because the greater welfare obtained during periods when

Table 4 Closed Loop Equilibrium Solution under Bertrand second-stage competition ($\theta_l = 0$) with capacity $x_i = 456.2$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2
p_l [€/MWh]	135.9	133.0	129.9	126.7	123.3	119.7	115.8	111.8	107.4	102.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2
p_l [€/MWh]	97.9	92.7	87.1	81.0	74.5	67.5	59.9	51.6	42.6	32.8

Table 5 Closed Loop Equilibrium Solution under Cournot second-stage competition ($\theta_l = 1/\alpha_l$) with capacity $x_i = 555.8$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	555.8	555.8	555.8	555.8	555.8	555.8	539.9	524.0	508.2	492.3
p_l [€/MWh]	111.1	107.5	103.7	99.8	95.6	91.2	91.2	91.2	91.2	91.2
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	476.4	460.5	444.6	428.8	412.9	397.0	381.1	365.2	349.4	333.5
p_l [€/MWh]	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2

Table 6 Closed Loop Equilibrium Solution assuming Allaz-Vila second-stage competition ($\theta_l = 1/(2\alpha_l)$) with capacity $x_i = 515.2$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2
p_l [€/MWh]	121.2	117.9	114.4	110.8	106.9	102.8	98.5	93.9	89.0	83.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	515.2	515.2	515.2	514.5	495.5	476.4	457.3	438.3	419.2	400.2
p_l [€/MWh]	78.3	72.3	66.0	59.4	59.4	59.4	59.4	59.4	59.4	59.4

capacity is slack (and Allaz-Vila prices are lower and closer to production cost) offsets the welfare loss during peak periods when the greater Cournot capacity yields lower prices.

Another surprise is that not only market efficiency but also profits are nonmonotonic in θ. Both Bertrand and Cournot profits are higher than Allaz-Vila profits; the lowest profit thus occurs when market efficiency is highest, at least under these parameters. However, higher profits do not always imply lower market efficiency, as a comparison of the Bertrand and Cournot cases shows. Cournot shows higher profit, consumer surplus, and market efficiency than Bertrand competition. That is, Cournot is Pareto superior to Bertrand in this because all parties are better off under the Cournot equilibrium.

Table 7 Market Efficiency (ME), Consumer Surplus (CS) and Total Profit in Closed Loop Solutions.

	Bertrand	Cournot	Allaz-Vila
ME [€]	$1.24 \cdot 10^9$	$1.28 \cdot 10^9$	$1.30 \cdot 10^9$
CS [€]	$0.621 \cdot 10^9$	$0.642 \cdot 10^9$	$0.717 \cdot 10^9$
Total Profit [€]	$0.621 \cdot 10^9$	$0.636 \cdot 10^9$	$0.584 \cdot 10^9$

6 Conclusions

In this paper we compare two types of models for modeling the generation capacity expansion game: an open loop model describing a game in which investment and operation decisions are made simultaneously, and a closed loop equilibrium model, where investment and operation decisions are made sequentially. In both models the market is represented via a conjectured price response, which allows us to capture various degrees of oligopolistic behavior. Setting out to characterize the differences between these two models, we have found that for one load period, the closed loop equilibrium equals the open loop Cournot equilibrium for any choice of conjectured price response between Bertrand and Cournot competition — a generalization of the finding of Kreps and Scheinkman [20]. In the case of multiple load periods, this result can be extended. In particular, if closed loop models under different conjectures have the same set of load periods in which capacity is constraining and the corresponding multipliers are positive, then their first stage capacity decisions are the same, although not their outputs during periods when capacity is slack. Furthermore, if the Cournot open and closed loop solutions have the same periods when capacity constrains, then their solutions are identical.

As indicated in the first numerical example, this indicates that when having market behavior close to Cournot competition, the additional effort of computing the closed loop model (as opposed to the simpler open loop model) does not pay off because the outcomes are either exactly the same or very similar depending on the data. But if behavior on the spot market is far from Cournot competition and approaching Bertrand competition, the additional modeling effort might be worthwhile, as the closed loop model is capable of depicting a feature that the open loop model fails to capture, which is that generation companies would not voluntarily build all the capacity that might be determined by the spot market equilibrium if that meant less profits for themselves. Thus the closed loop model could be useful to evaluate the effect of alternative market designs for mitigating market power in spot markets and incenting capacity investments in the long run, e.g. capacity mechanisms, in Sakellaris [32]. Extensions could also consider the effect of forward energy contracting as well (as in Murphy and Smeers [26]). These policy analyses will be the subject of future research.

The second numerical example demonstrates that depending on the choice of parameters, more competition in the spot market may lead to less market efficiency and less consumer surplus in the closed loop model. This surprising result indicates that regulatory approaches that encourage or mandate marginal pricing in the spot market in order to protect consumers may actually lead to situations in which both consumers and generation companies are worse off.

In future research we will address the issue of existence and uniqueness of solutions, as has been done for the Cournot case by Murhphy and Smeers [25], who found that a pure-strategy closed loop equilibrium does not necessarily exist but if it exists it is unique. Moreover, the games presented here will be extended to multi-year games with sequential capacity decisions, and the effects of forward contracting will be investigated.

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